Formally Packing 8-Dimensional Spheres

ItaLean 2025, Bologna

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11 December, 2025

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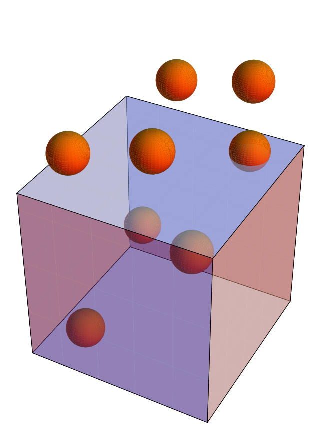

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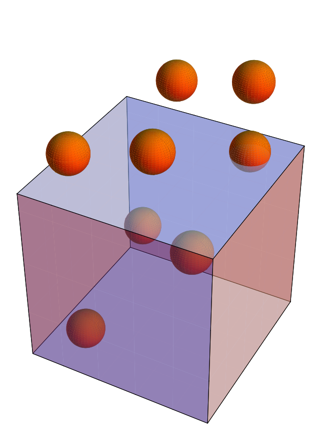

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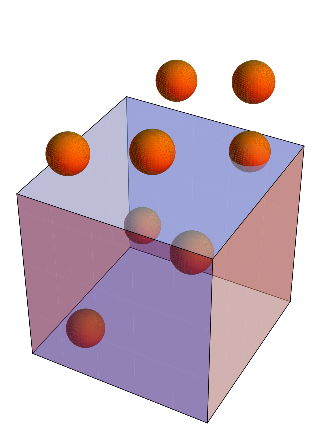

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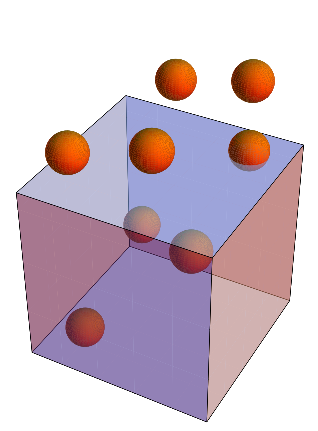

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The sphere packing problem asks for a densest configuration of non-overlapping equally sized balls in the d-dimensional Euclidean space.

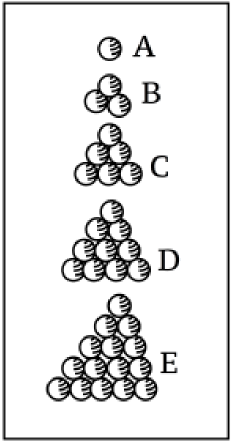
The maximal possible density of such a configuration in d-dimensional space is called the sphere packing constant and is denoted by Δ_d.

What is known about Δ_d?

$\Delta_1 = 1$

$\Delta_2 = \frac{\pi}{\sqrt{12}} \approx 0.9069$

$\Delta_3 = \frac{\pi}{\sqrt{18}} \approx 0.7405$



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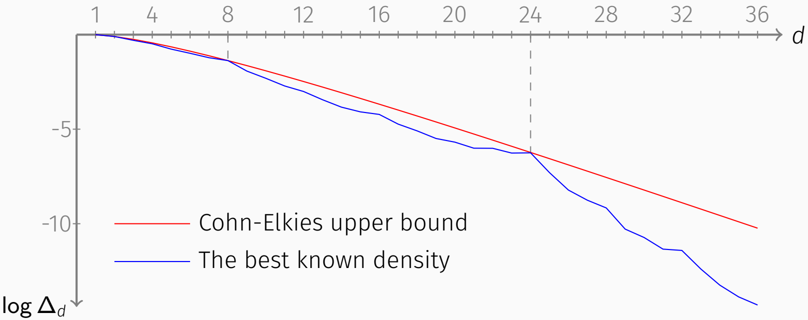

Dimension 8

E_8-lattice $\Lambda_8 \subset \mathbb{R}^8$ is given by

$$\Lambda_8 = \{(x_i) \in \mathbb{Z}^8 \cup (\mathbb{Z} + \tfrac{1}{2})^8 | \ \textstyle\sum_{i=1}^8 x_i \equiv 0 \pmod 2\}.$$

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Theorem (V. 2016)
No packing of unit balls in Euclidean space $\mathbb{R}^8$ has density greater than that of the E_8-lattice packing. Therefore $\Delta_8 = \frac{\pi^4}{384} \approx 0.25367$.

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Theorem (Cohn, Kumar, Miller, Radchenko, V. 2016)
No packing of unit balls in Euclidean space $\mathbb{R}^{24}$ has density greater than that of the Λ_{24}-lattice packing. Therefore $\Delta_{24} = \frac{\pi^{12}}{12!} \approx 0.00193$.

Theorem (Cohn, Elkies 2003, independently Gorbatchev 2000's)
Suppose that $r_0 > 0$ and $f : \mathbb{R}^d \to \mathbb{R}$ is a Schwartz function such that:

$$f(x) \leq 0 \text{ for } \|x\| \geq r_0$$

$$\widehat{f}(x) \geq 0 \text{ for all } x \in \mathbb{R}^d$$

$$f(0) = \widehat{f}(0) = 1.$$

Then

$$\Delta_d \leq \mathrm{Vol}(B_d(0, r_0/2)).$$

Proof

Let $X \subset \mathbb{R}^d$ and $\|x - y\| \geq r_0$ for any distinct $x, y \in X$

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$$\geq \sum_{x \in X} \sum_{y \in X/L} f(x-y) = \sum_{x \in X/L} \sum_{y \in X/L} \sum_{\ell \in L} f(x - y + \ell)$$

use Poisson summation formula

$$= \sum_{x \in X/L} \sum_{y \in X/L} \frac{1}{\mathrm{vol}(\mathbb{R}^d/L)} \sum_{m \in L^*} \widehat{f}(m)\, e^{2\pi i m(x-y)}$$

$$= \frac{1}{\mathrm{vol}(\mathbb{R}^d/L)} \sum_{m \in L^*} \widehat{f}(m) \cdot \Big| \sum_{x \in X/L} e^{2\pi i m x} \Big|^2 \geq$$

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$$\geq \frac{\sharp(X/L)^2}{\mathrm{vol}(\mathbb{R}^d/L)} \cdot \widehat{f}(0).$$

Theorem (Cohn, Elkies 2003)

$$\Delta_8 \leq 1.00016\, \Delta_{E_8},$$

$$\Delta_{24} \leq 1.019\, \Delta_{\Lambda_{24}}.$$

Theorem (V 2016)
There exists a radial Schwartz function $f_{E_8} : \mathbb{R}^8 \to \mathbb{R}$ which satisfies:

$$f_{E_8}(x) \leq 0 \text{ for } \|x\| \geq \sqrt{2}$$
$$\widehat{f}_{E_8}(x) \geq 0 \text{ for all } x \in \mathbb{R}^8$$
$$f_{E_8}(0) = \widehat{f}_{E_8}(0) = 1.$$

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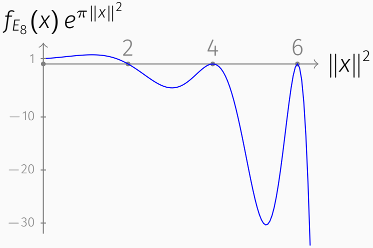
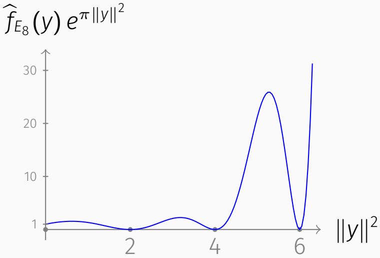
Plot of the functions f_{E_8} and $\widehat{f}_{E_8}$





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By the Poisson summation formula we have

$$f_{E_8}(0) \geq \sum_{\ell \in \Lambda_8} f_{E_8}(\ell) = \sum_{\ell \in \Lambda_8} \widehat{f}_{E_8}(\ell) \geq \widehat{f}_{E_8}(0).$$

This can happen only if $f_{E_8}(\sqrt{2n}) = \widehat{f}_{E_8}(\sqrt{2n}) = 0$ for all $n \in \mathbb{Z}_{>0}$.

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Remark 3
We have constructed the function f_{E_8} in the form

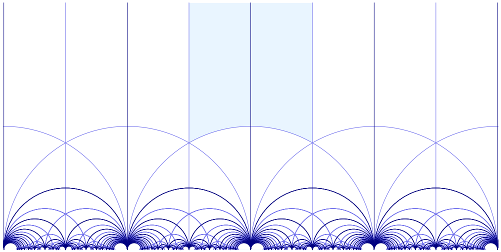
$$f_{E_8}(r) = \sin(\pi r^2/2)^2 \int_0^\infty \varphi(it)\, e^{-\pi r^2 t} dt$$

where φ is a holomorphic function in the upper half-plane.

Hidden symmetry

Let $\mathbb{H}$ be the upper half-plane $\{z \in \mathbb{C} | \operatorname{Im}(z) > 0\}$. Consider the modular group $\Gamma_1 := \mathrm{PSL}_2(\mathbb{Z})$. The group Γ_1 acts on $\mathbb{H}$ by linear fractional transformations

$$\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) z := \frac{az+b}{cz+d}.$$



A *(holomorphic) modular form* of weight k and a finite index subgroup $\Gamma \subseteq \Gamma_1$ is a holomorphic function $f : \mathbb{H} \to \mathbb{C}$ such that:

1. $f\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f(z)$ for all $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma$
2. for each $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma_1$ $(cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right)$ has the Fourier expansion $(cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right) = \sum_{n=0}^{\infty} c_\alpha(n)\, e^{2\pi i \frac{n}{h} z}$ for some $h \in \mathbb{N}$.

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Let $M_k(\Gamma)$ be the space of modular forms of weight k and a subgroup Γ. Spaces $M_k(\Gamma)$ are finite dimensional.

A *weakly-holomorphic modular form* of weight k and a finite index subgroup $\Gamma \subseteq \Gamma_1$ is a holomorphic function $f : \mathbb{H} \to \mathbb{C}$ such that:

1. $f\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f(z)$ for all $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma$
2. for each $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma_1$ $(cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right)$ has the Fourier expansion $(cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right) = \sum_{n \gg -\infty}^{\infty} c_\alpha(n)\, e^{2\pi i \frac{n}{h} z}$ for some $h \in \mathbb{N}$.

We denote the space of weakly-holomorphic modular forms of weight k and group Γ by $M_k^!(\Gamma)$. The spaces $M_k^!(\Gamma)$ are infinite dimensional.

For an even integer $k \geq 4$ we define *weight k Eisenstein series* as

$$E_k(z) := \frac{1}{2\zeta(k)} \sum_{(c,d)\in\mathbb{Z}^2\setminus(0,0)} (c\tau + d)^{-k}. \tag{1}$$

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Since the sum converges absolutely, it is easy to see that $E_k \in M_k(\Gamma_1)$.

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Since the sum converges absolutely, it is easy to see that $E_k \in M_k(\Gamma_1)$. Eisenstein series possesses the Fourier expansion

$$E_k(z) = 1 + \frac{2}{\zeta(1-k)} \sum_{n=1}^{\infty} \sigma_{k-1}(n)\, e^{2\pi i z}, \tag{2}$$

where $\sigma_{k-1}(n) = \sum_{d|n} d^{k-1}$.

Example 1. Eisenstein series

Infinite sum (1) does not converge absolutely for $k = 2$. On the other hand, the expression (2) converges to a holomorphic function on the upper half-plane and therefore we set

$$E_2(z) := 1 - 24 \sum_{n=1}^{\infty} \sigma_1(n)\, e^{2\pi i n z}.$$

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Weight two Eisenstein series E_2 is a *quasimodular form.*

Example 2. Theta series

$$
\begin{aligned}
\theta_{00}(z) &= \sum_{n\in\mathbb{Z}} e^{\pi i n^2 z} \\
\theta_{01}(z) &= \sum_{n\in\mathbb{Z}} (-1)^n \, e^{\pi i n^2 z} \\
\theta_{10}(z) &= \sum_{n\in\mathbb{Z}} e^{\pi i (n+\frac{1}{2})^2 z}.
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Modularity of theta series follows from *Poisson summation formula*.

Probably, the most famous wealky-holomorphic modular form is the *elliptic j-invariant*

$$j \;:=\; \frac{1728\, E_4^3}{E_4^3 - E_6^2}.$$

This function belongs to $M_0^!(\Gamma_1)$ and has the Fourier expanssion

$$j(z) \;=\; q^{-1} + 744 + 196884q + 21493760q^2 + O(q^3)$$

where $q = e^{2\pi i z}$.

Consider $\phi \in M_0^{!,\mathrm{quasi}}(\Gamma_1)$ and $\psi \in M^!_{-2}(\Gamma(2))$

$$\phi = 518400q + 31104000q^2 + 870912000q^3 + 15697152000q^4 + O(q^6)$$

$$\psi = q^{-1} + 144 - 5120q^{1/2} + 70524q - 626688q^{3/2} + O(q^2)$$

Explicit solution of the Cohn-Elkies linear prog. problem

Consider $\phi \in M_0^{!,\text{quasi}}(\Gamma_1)$ and $\psi \in M_{-2}^{!}(\Gamma(2))$

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$$g(r) := \frac{-\pi}{2160}\,\sin(\pi r^2/2)^2 \int_0^\infty \left(t^2\,\phi(i/t) + \frac{36}{\pi^2}\psi(it)\right)\, e^{-\pi r^2 t}\,dt$$

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Theorem
The radial function $g : \mathbb{R}^8 \to \mathbb{R}$ satisfies:

$$\begin{aligned}&g(x) \leq 0 \text{ for } \|x\| \geq \sqrt{2}\\ &\widehat{g}(x) \geq 0 \text{ for all } x \in \mathbb{R}^8\\ &g(0) = \widehat{g}(0) = 1.\end{aligned}$$

Step 1

$$\widehat{g}(r) = \frac{\pi}{2160}\,\sin(\pi r^2/2)^2 \int_0^\infty \left(-t^2\,\phi(i/t) + \frac{36}{\pi^2}\psi(it)\right) e^{-\pi r^2 t}\,dt$$

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Step 2

$$-t^2\,\phi(i/t) - \tfrac{36}{\pi^2}\psi(it) < 0 \quad \text{for } t \in (0,\infty)$$

Proof

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$$-t^2\,\phi(i/t) - \tfrac{36}{\pi^2}\psi(it) < 0 \quad \text{for } t \in (0,\infty)$$

Step 3

$$-t^2\,\phi(i/t) + \tfrac{36}{\pi^2}\psi(it) > 0 \quad \text{for } t \in (0,\infty)$$

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Step 4

$g(0) = \widehat{g}(0) = 1.$

Progress on the Formalisation

Formalising Sphere Packing in Lean

by Chris Birkbeck, Sidharth Hariharan, Gareth Ma, Bhavik Mehta, Seewoo Lee, Maryna Viazovska

Blueprint (web) | Blueprint (pdf) | Documentation | GitHub

Formalising Sphere Packing in Lean

In 2022, Maryna Viazovska was awarded the Fields Medal for solving the sphere packing problem in dimension 8. This project formalises this result in the Lean Theorem Prover following her original paper and followup calculations by Lee. It is an online, open-source collaboration currently being led by Christopher Birkbeck, Sidharth Hariharan, Bhavik Mehta and Seewoo Lee. If you'd like to contribute, you may find the following links useful!

- Zulip chat for coordination
- Blueprint
- Blueprint as pdf
- Dependency graph
- Doc pages for this repository
- Project dashboard
- Viazovska's original paper

SCAN TO VISIT OUR WEBSITE!

`https://thefundamentaltheor3m.github.io/Sphere-Packing-Lean`

A Brief History of Sphere Packing in Lean

`https://thefundamentaltheor3m.github.io/Sphere-Packing-Lean`

1. March 2024: Viazovska and I kickstart project in Lausanne.
2. July-September 2024:
   - Ma and I formalise basic theory.
   - I formalise much of Cohn-Elkies.
   - Birkbeck and Lee begin formalising facts about modular forms.
3. October 2024-June 2025:
   - I work on formalising Viazovska's magic function for my Master's project at Imperial College London under Bhavik Mehta.
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4. 13 June, 2025: Viazovska unveils project to public at Big Proof 2025.
5. 19 October, 2025: First packathon!

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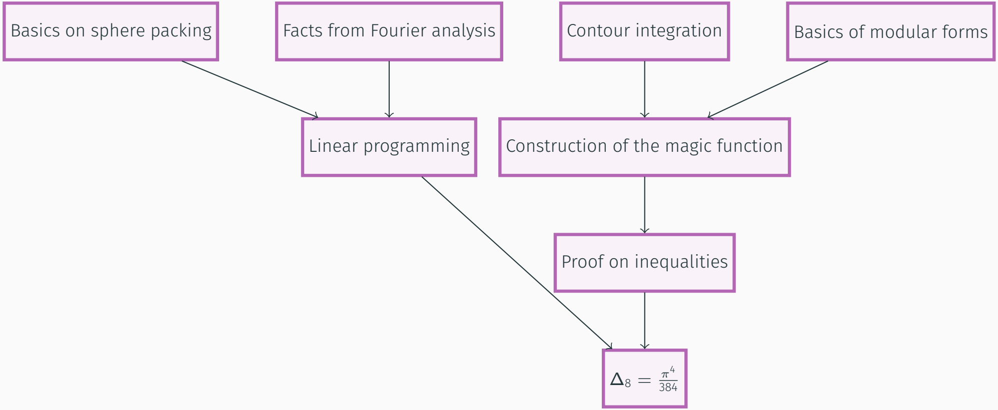
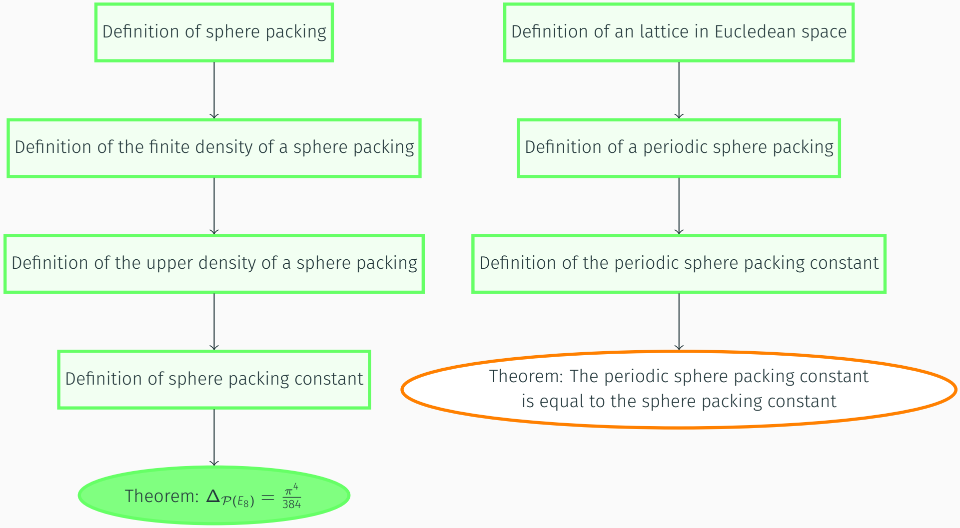
Packathons are Mondays 11 am Eastern Time. They resume in January. See Zulip for updates!

Blueprint of a blueprint



Sphere Packing Basics



Fourier analysis and Poisson summation formula

Definition of the Fourier Transform

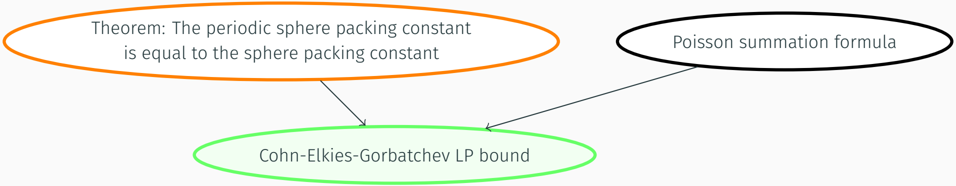
Definition of the Schwartz functions

Lemma: The Fourier transform is a continous,
linear automorphism of the space of Schwartz functions

Theorem (Poisson summation formula) :
Let $d > 0$ and let $\Lambda \subset \mathbb{R}^d$ be a lattice.
Let $f : \mathbb{R}^d \to \mathbb{R}$ be a Schwartz function.
For all vectors $v \in \mathbb{R}^d$, we have

$$\sum_{\ell \in \Lambda} f(\ell + v) = \frac{1}{\mathrm{Vol}(\mathbb{R}^d/\Lambda)} \sum_{m \in \Lambda^*} \widehat{f}(m) e^{-2\pi i \langle v, m \rangle}$$

Formalizing Cohn-Elkies-Gorbatchev linear programming bound



The following version of the Cauchy-Goursat had been formalised prior to this project.

Cauchy-Goursat for Rectangles
Suppose $f : \mathbb{C} \to \mathbb{C}$ is a function such that $f(z) \to 0$ as $\mathrm{Im}(z) \to \infty$. Then, for all $x_1, y_1, x_2, y_2 \in \mathbb{R}$, if f is holomorphic at all but countably many $z \in \mathbb{C}$ with $x_1 < \mathrm{Re}(z) < x_2$ and $y_1 < \mathrm{Im}(z) < y_2$ and continuous on the corresponding closed rectangle, then

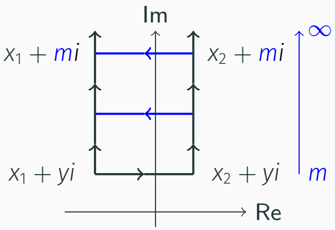
$$\int_{x_1}^{x_2} f(x + y_1 i)\,\mathrm{d}x - \int_{x_1}^{x_2} f(x + y_2 i)\,\mathrm{d}x + i\int_{y_1}^{y_2} f(x_2 + yi)\,\mathrm{d}y - i\int_{y_1}^{y_2} f(x_1 + yi)\,\mathrm{d}y = 0$$

We have adapted this to the following unbounded version.

Theorem(Cauchy-Goursat for Unbounded Rectangle-like Contours)
Suppose $f : \mathbb{C} \to \mathbb{C}$ is a function such that $f(z) \to 0$ as $\mathbf{Im}(z) \to \infty$. Then, for all $x_1, x_2, y \in \mathbb{R}$, if f is holomorphic at all but countably many $z \in \mathbb{C}$ with $x_1 < \mathbf{Re}(z) < x_2$ and $y < \mathbf{Im}(z)$ and then

$$\int_y^{\infty} f(x_1 + ti)\,\mathrm{d}t = \int_{x_1}^{x_2} f(t + yi)\,\mathrm{d}t + \int_y^{\infty} f(x_2 + ti)\,\mathrm{d}t.$$

provided that the integrals along the vertical contours exist.

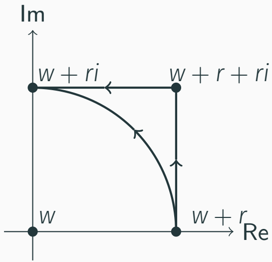


Theorem(Cauchy-Goursat: Squares and Circles)
Fix $w \in \mathbb{C}$ and $r > 0$. Let γ be the quarter-circle parameterized by $\gamma(t) = w + r\cos(t) + ri\sin(t)$ for $0 \leq t \leq \pi/2$. Let

$$S = \{x + yi \in \mathbb{C} \mid (x^2 + y^2 > r^2) \wedge (0 < x < r) \wedge (0 < y < r)\}.$$

For any $f : \mathbb{C} \to \mathbb{C}$ that is holomorphic on all but countably many points in S and continuous on the closure $\bar{S}$ of S,

$$\int_\gamma f(z)\,\mathrm{d}(z) = \int_{w+r}^{w+r+ir} f(z)\,\mathrm{d}(z) + \int_{w+r+ir}^{w+ir} f(z)\,\mathrm{d}(z).$$



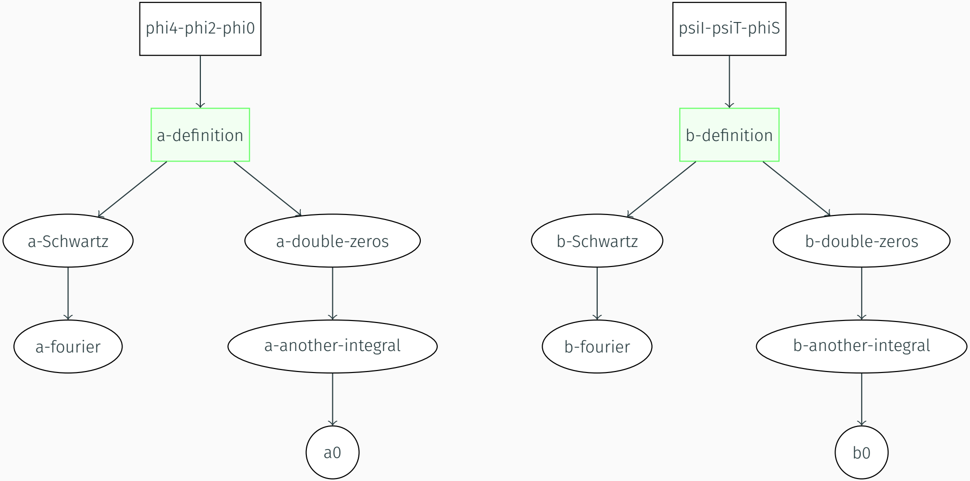
Theorem
Let $U \subset \mathbb{C}$ be a convex open subset of $\mathbb{C}$. If $f : U \to \mathbb{C}$ is holomorphic on all points in U, then there exists a holomorphic function $F : U \to \mathbb{C}$ such that $F'(z) = f(z)$ for all $z \in U$.

Theorem
Let $U \subset \mathbb{C}$, $f : U \to \mathbb{C}$ and $F : U \to \mathbb{C}$ be as above. Let $\gamma : [0, 1] \to U$ be a smooth path. Then

$$\int_\gamma f(z)\,\mathrm{d}(z) = F(\gamma(1)) - F(\gamma(0)).$$

These theorems are already formalized in Isabelle. Working together with Auguste Poiroux we have formalized the first theorem above in Lean (for now for U equal to the unit disk).

Construction of the magic function



Proving inequalities

$$A(t) := -t^2\,\phi(i/t) - \tfrac{36}{\pi^2}\psi(it) < 0 \quad \text{for } t \in (0,\infty)$$

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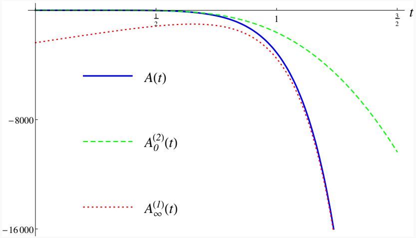

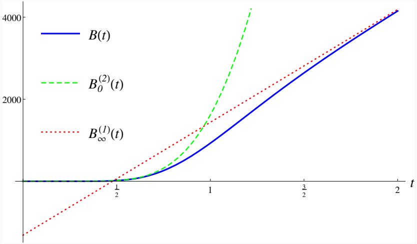

Original approach: approximation by truncated Fourier expansion and applying interval arithmetic.

Proving inequalities

Analytic proof by Dan Romik

ON VIAZOVSKA'S MODULAR FORM INEQUALITIES

DAN ROMIK

Abstract. Viazovska proved that the E_8 lattice sphere packing is the densest sphere packing in 8 dimensions. Her proof relies on two inequalities between functions defined in terms of modular and quasimodular forms. We give a direct proof of these inequalities that does not rely on computer calculations.

Analytic proof by Seewoo Lee

Algebraic proof of modular form inequalities for optimal sphere packings

Seewoo Lee

Abstract. We give algebraic proofs of Viazovska and Cohn-Kumar-Miller-Radchenko-Viazovska's modular form inequalities for 8 and 24-dimensional optimal sphere packings.

Can Machines Pack Spheres?

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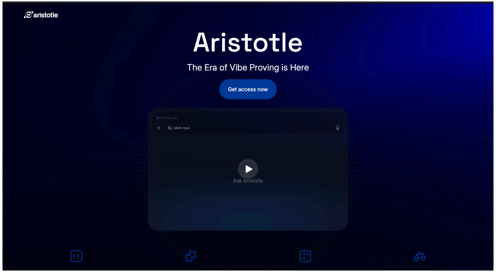

Article | Published: 12 November 2025

Olympiad-level formal mathematical reasoning with reinforcement learning

Thomas Hubert, Rishi Mehta, Laurent Sartran, Miklós Z. Horváth, Goran Žužić, Eric Wieser, Aja Huang, Julian Schrittwieser, Yannick Schroecker, Hussain Masoom, Ottavia Bertolli, Tom Zahavy, Amol Mandhane, Jessica Yung, Iuliya Beloshapka, Borja Ibarz, Vivek Veeriah, Lei Yu, Oliver Nash, Paul Lezeau, Salvatore Mercuri, Calle Sönne, Bhavik Mehta, Alex Davies, Daniel Zheng, Fabian Pedregosa, Yin Li, Ingrid von Glehn, Mark Rowland, Samuel Albanie, Ameya Velingker, Simon Schmitt, Edward Lockhart, Edward Hughes, Henryk Michalewski, Nicolas Sonnerat, Demis Hassabis, Pushmeet Kohli & David Silver

— Show fewer authors

AI-Written Proofs Exploiting Bugs

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```
set_option linter.style.longLine false
set_option maxHeartbeats 500000 in
lemma iteratedDeriv_comp_sq (g : ℝ → ℝ) (hg : ContDiff ℝ ∞ g) (k : ℕ) :
    iteratedDeriv (2 * k) (fun x => g (x ^ 2)) 0 = ((Nat.factorial (2 * k) : ℝ) / (Nat.factorial k : ℝ)) * iteratedDeriv k g 0 := by
  -- By definition of $g$, we know that its Taylor expansion at 0 is given by the Taylor series of $g$.
  have h_taylor : ∀ x, g x = ∑ n ∈ Finset.range (k + 1), deriv^[n] g 0 / n ! * x ^ n + x ^ (k + 1) * ∫ t in (0 : ℝ)..1, (1 - t) ^ k / k ! * deriv^[k
    -- Apply the Taylor series expansion theorem to g at 0.
    have h_taylor : ∀ x, g x = ∑ n ∈ Finset.range (k + 1), deriv^[n] g 0 / n ! * x ^ n + ∫ t in (0 : ℝ)..x, (x - t) ^ k / k ! * deriv^[k + 1] g t :=
      induction' k with k ih;
      · aesop;
        rw [ intervalIntegral.integral_deriv_eq_sub ] <;> norm_num [ hg.contDiffAt.differentiableAt ];
        -- Since $g$ is infinitely differentiable, its derivative $g'$ is continuous.
        have h_cont_diff : Continuous (deriv g) := by
          simpa using hg.continuous_deriv;
        exact h_cont_diff.intervalIntegrable _ _;
      · -- Apply the integration by parts formula to the integral.
        have h_parts : ∀ a b x, ∫ t in a..b, (x - t) ^ (k + 1) / (Nat.factorial (k + 1)) * deriv^[k + 2] g t = (x - b) ^ (k + 1) / (Nat.factorial (k
          intro a b x;
          rw [ intervalIntegral.integral_mul_deriv_eq_deriv_mul ];
          rotate_left;
          -- The derivative of $(x - t)^{k+1} / (k+1)!$ with respect to $t$ is $-(x - t)^k / k!$.
          have h_deriv : ∀ t, HasDerivAt (fun t => (x - t) ^ (k + 1) / (Nat.factorial (k + 1))) (-(x - t) ^ k / (Nat.factorial k)) t := by
            -- Apply the chain rule to compute the derivative.
            intro t
            have h_chain : HasDerivAt (fun t => (x - t) ^ (k + 1)) (-(k + 1) * (x - t) ^ k) t := by
              -- Apply the chain rule to compute the derivative: the derivative of $(x - t)^{k+1}$ with respect to $t$ is $-(k+1)(x - t)^k$.
              have h_chain : HasDerivAt (fun t => (x - t)) (-1) t := by
                exact hasDerivAt_id t |> HasDerivAt.const_sub x;
              convert h_chain.pow ( k + 1 ) using 1 ; norm_num ; ring;
            convert h_chain.div_const ( ( k + 1 ) ! : ℝ ) using 1 ; push_cast [ Nat.factorial_succ ] ; ring;
            -- Combine and simplify the fractions
            field_simp
            ring;
          exact?;
          rotate_left;
```

AI-Written Proofs Exposing Bugs

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AI-Written Proofs Exposing Bugs

Another unfortunate thing about AI models is that they are too trusting of humans. I'm flattered they think I'm always right, but I'd like to be told when I'm wrong. See PR #181 (Gauss).

```
theorem J₃'_smooth' : ContDiff ℝ ∞ RealIntegrals.J₃' := by
  have hJ : RealIntegrals.J₃' = fun x : ℝ => cexp (2 * π * I * x) * RealIntegrals.J₁' x := by
    funext x
    have hEqOn : EqOn (fun t : ℝ => I * ψT' (z₁' t) * cexp (π * I * x * (z₃' t)))
        (fun t : ℝ => cexp (2 * π * I * x) * (I * ψT' (z₁' t) * cexp (π * I * x * (z₁' t))))
        (uIcc (0 : ℝ) 1) := by
      intro t ht
      have ht' : t ∈ Icc (0 : ℝ) 1 := by simpa [uIcc_of_le (by norm_num : (0 : ℝ) ≤ 1)] using ht
      have hz32 : z₃' t = z₁' t + (2 : ℂ) := by
        have h : z₃' t - z₁' t = (2 : ℂ) := by
          simp [z₁'_eq_of_mem ht', z₃'_eq_of_mem ht', one_add_one_eq_two]
        simpa [add_comm] using
          (sub_eq_iff_eq_add' (a := z₃' t) (b := z₁' t) (c := (2 : ℂ))).1 h
      simp [hz32, mul_add, Complex.exp_add, mul_comm, mul_left_comm, mul_assoc]
    simpa [RealIntegrals.J₃', RealIntegrals.J₁'] using
      (intervalIntegral.integral_congr (a := (0 : ℝ)) (b := (1 : ℝ)) hEqOn).trans
        (by
          simp [mul_comm, mul_left_comm, mul_assoc])
  have h_exp : ContDiff ℝ ∞ (fun x : ℝ => cexp (2 * π * I * x)) := by
    have hmul : ContDiff ℝ ∞ (fun x : ℝ => (2 * π * I : ℂ) * (x : ℂ)) :=
      contDiff_const.mul (by simpa using (ofRealCLM.contDiff : ContDiff ℝ ∞ (fun x : ℝ => (x : ℂ))))
    simpa [mul_comm, mul_left_comm, mul_assoc] using hmul.cexp
  simpa [hJ] using (h_exp.mul J₁'_smooth')
```

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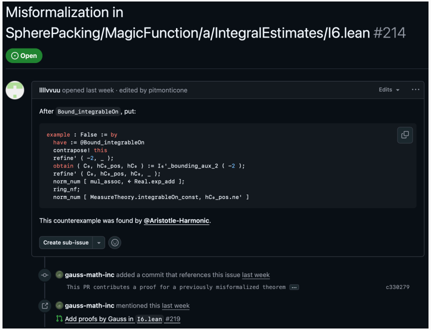

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Misformalization in SpherePacking/MagicFunction/a/IntegralEstimates/I6.lean #214

Open

IIIlvvuu opened last week · edited by pitmonticone

Edits

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Create sub-issue

gauss-math-inc added a commit that references this issue last week

This PR contributes a proof for a previously misformalized theorem … c330279

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Add proofs by Gauss in `I6.lean` #219

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lllvvuu opened last week · edited by pitmonticone

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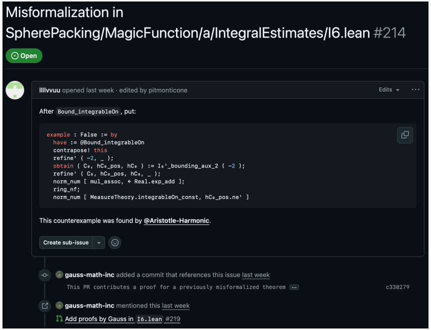
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  norm_num [ mul_assoc, ← Real.exp_add ];
  ring_nf;
  norm_num [ MeasureTheory.integrableOn_const, hC₀_pos.ne' ]
```

This counterexample was found by @Aristotle-Harmonic.

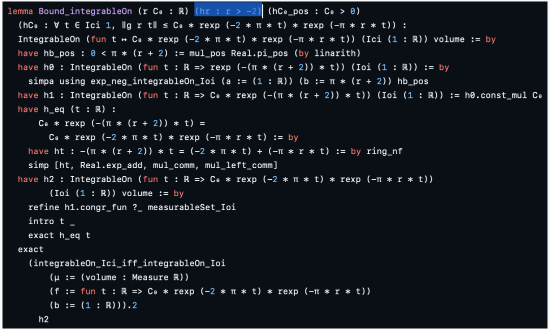
Create sub-issue

gauss-math-inc added a commit that references this issue last week

This PR contributes a proof for a previously misformalized theorem … c330279

gauss-math-inc mentioned this last week

Add proofs by Gauss in I6.lean #219



```
lemma Bound_integrableOn (r C₀ : ℝ) (hr : r > -2) (hC₀_pos : C₀ > 0)
  (hC₀ : ∀ t ∈ Ici 1, ‖g r t‖ ≤ C₀ * rexp (-2 * π * t) * rexp (-π * r * t)) :
  IntegrableOn (fun t ↦ C₀ * rexp (-2 * π * t) * rexp (-π * r * t)) (Ici (1 : ℝ)) volume := by
  have hb_pos : 0 < π * (r + 2) := mul_pos Real.pi_pos (by linarith)
  have h0 : IntegrableOn (fun t : ℝ => rexp (-(π * (r + 2)) * t)) (Ioi (1 : ℝ)) := by
    simpa using exp_neg_integrableOn_Ioi (a := (1 : ℝ)) (b := π * (r + 2)) hb_pos
  have h1 : IntegrableOn (fun t : ℝ => C₀ * rexp (-(π * (r + 2)) * t)) (Ioi (1 : ℝ)) := h0.const_mul C₀
  have h_eq (t : ℝ) :
      C₀ * rexp (-(π * (r + 2)) * t) =
        C₀ * rexp (-2 * π * t) * rexp (-π * r * t) := by
    have ht : -(π * (r + 2)) * t = (-2 * π * t) + (-π * r * t) := by ring_nf
    simp [ht, Real.exp_add, mul_comm, mul_left_comm]
  have h2 : IntegrableOn (fun t : ℝ => C₀ * rexp (-2 * π * t) * rexp (-π * r * t))
        (Ioi (1 : ℝ)) volume := by
    refine h1.congr_fun ?_ measurableSet_Ioi
    intro t _
    exact h_eq t
  exact
    (integrableOn_Ici_iff_integrableOn_Ioi
        (μ := (volume : Measure ℝ))
        (f := fun t : ℝ => C₀ * rexp (-2 * π * t) * rexp (-π * r * t))
        (b := (1 : ℝ))).2
      h2
```

The reason this statement was misformalised is that r is meant to be the norm of a vector. If a human had worked on this proof, I'm sure they'd have spotted it too. However, I seriously doubt their fix would have been to assume $r > -2$...

The Bottom Line

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Questions?